

Discrete Time-Frequency Signal Analysis and Processing Techniques for ECG Signals

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Abstract - Nonstationary ECG signal was analyzed using time-frequency distributions to enhance diagnosis techniques. The time-frequency distributions were formulated using discrete Wigner-Ville distribution with various windowing techniques. The discrete pseudo Wigner-Ville distribution (DPWVD) and discrete smoothed Wigner-Ville distribution (DSWVD) were formed to analyze supra-ventricular and malignant ventricular ECG arrhythmia signals. The performance of the time-frequency distributions was evaluated of energy distribution and time-frequency resolution only. The result reveals that discrete Wigner-Ville distribution computed in time lag domain introduced cross-term. The discrete pseudo Wigner-Ville distribution was computed in time lag domain and provided low time-frequency resolution. The discrete smoothed Wigner-Ville distribution window was computed in time and Doppler lag domain in which the Doppler lag domain found satisfactory performance compare to DWVD and DPWVD.

Index Terms - Discrete Pseudo Wigner-Ville Distribution, Discrete Smoothed Wigner-Ville Distribution, Discrete Wigner-Ville Distribution, ECG arrhythmia, Time-Frequency Distribution

1 INTRODUCTION

Most general, most of the real-life signals are nonstationary, multicomponent signals described in [1]. The electrical activity of the heart system generates a pattern of ECG signal during atrial depolarization, ventricular depolarization, and ventricular repolarization and these are represented as P-QRS-T complex wave respectively. The shape, relative position, duration, and amplitude of these waves are considered an important diagnostic tool to a cardiologist in the diagnostic process described in [2]. Including an ECG signal biomedical signals are characterized as time-varying signal properties, they are nonstationary where the signal components with time-varying properties occur at different frequencies in the ECG signal. Hence classical methods are not suitable to analyze the time-varying characteristic signal. Therefore, the time-variant frequency-selective approach is required for their "time-frequency" analysis [3]. Time-frequency techniques are found more suitable it maps the one-dimensional time-domain signal into two-dimensional time-frequency representation [4]. They describe signal energy around the instantaneous frequency both on time-frequency space [5]. The Wigner-Ville distribution is an important algorithm of time-frequency analysis in biomedical signal processing [6], [7]. It has the best time-frequency resolution properties described in [8],[9],[10],[11]. The bilinear nature of Wigner-Ville distribution introduced a cross-term for a multicomponent, nonstationary signal but it preserves most of its properties hence so many researchers are investigated to resolve this problem long time [12],[13],[14].

Wigner-Ville distribution is a primary distribution to form so many classes of bilinear distribution in which windowed Wigner-Ville distribution is one of the recent development time-frequency distribution [15]. The research extends to the work of the analysis of the ECG signal based on windowed Wigner-Ville distribution. Some of the windowing techniques proposed to formulate time-frequency representation are discrete pseudo-winger-Ville distribution, smoothed pseudo-Wigner-Ville distribution with Lag independent window and Doppler independent window. The discrete form of time-

frequency distributions is computed with hanning, hamming and Gauss window in time lag domain and Doppler lag domain to analyze the supra-ventricular, malignant ventricular arrhythmia ECG signals.

This paper is organized as chapter 2 presents ECG signal and its characteristics, analytical signal and signal model of the window characteristics. The methodology to form time-frequency distribution is discussed in section 3. The simulation results and discussion are present in section 4.

2 ECG Arrhythmia signal

The up normal electrical activity of the heart causes an arrhythmia may be cardiovascular disease. It is a defect to conduct the electrical impulses from the right article to AV node and AV node to right ventricle at that time the heartbeat may be too fast, too slow or maybe regular or irregular. The abnormal electrical activity of the heart broadly classified as supraventricular arrhythmia and malignant ventricular arrhythmia [16]. In this analysis, supraventricular arrhythmia and malignant ventricular arrhythmia ECG signals were tracked. A supraventricular arrhythmia occurs in the right article due to abnormal impulses arising from the atria. It has irregular shapes of QRS complexes [17]. Malignant ventricular arrhythmia occurs from AV node or ventricle. The QRS complexes are wide and where the T wave disappears [18]. The QRS complexes of the abnormal have irregular shapes and changes over time.

The ECG arrhythmia signals are obtained from the MIT BIH arrhythmia database [19]. The signals sampled at 360Hz. This time-domain signal is represented by $x[n]$ is the real, causal and bandlimited signal. It has both positive and negative frequency components introducing aliasing. The aliasing can be avoided by a technique called analytical signal.

2.1 Analytical Signal and Hilbert Transformation

To get an alias-free signal the real-valued signal $x[n]$ is converted into an analytical complex-valued signal using Hilbert transform defined in the time domain is

$$z[n] = x[n] + jH(x[n]) \quad (1)$$

where $H(x[n])$ is the Hilbert transform of the real-valued signal $x[n]$ and $z[n]$ is the analytical signal of the analytical associate $x[n]$. The negative frequency components are made zero, the analytical signal only contains positive frequency components by the way aliasing is avoided with the help of Hilbert transform.

2.2 Signal Modelling

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Before analyzing any signal using time-frequency distribution one can find whether the signal is monocomponent or multi-component signal and form a model of the signal for analysis. After calculating the analytical signal, the signal model is derived by extracting instantaneous amplitude (IA), instantaneous phase (IP) instantaneous frequency (IF) and group delay (ID). The bandwidth and time spread of the ECG arrhythmia are calculated and determined the ECG arrhythmia signal is a multicomponent signal and it is modeled as AM-FM signal model as given in equation (2)

$$z(n) = \sum_{k=1}^{N_c} z_k = \sum_{k=1}^{N_c} a_k(n) \cos(\varphi_k(n)) \quad (2)$$

3 Formulations of Time-Frequency Distributions

The Wigner-Ville distribution and a smoothing window formulated the time-frequency distribution to realize and implement in hardware and software the discrete version of the equation is formulated as

$$\rho[n, k] = 2 \underset{n \rightarrow k}{DFT} \left\{ G[n, m]_n^* (z[n+m]z^*[n-m]) \right\} \quad (3)$$

where n is time index, m is lag index, k is frequency index, z[n] is an analytical signal of analytical associate x[n], G[n, m] is the window function in time lag domain and ρ[n, k] is time-frequency distribution in the time-frequency domain.

$$k_z[n, m] = z[n+m]z^*[n-m] \quad (4)$$

where equation (4) is an instantaneous auto-correlation function(IACF) in time lag domain

3.1 Discrete Wigner-Ville Distribution

To form the discrete Wigner-Ville distribution the window function in the equation (3) is equal to a rectangular window therefore

$$G[n, m] = \delta[n] = 1 \quad (5)$$

only the instantaneous autocorrelation function (IACF) itself formed the Wigner-Ville distribution and it is given as

$$DWVD[n, k] = 2 \sum_{m=-\infty}^{\infty} z[n+m]z^*[n-m]e^{-j\frac{2\pi}{N}km} \quad (6)$$

The instantaneous autocorrelation is performed on the analytical signal z[n] and its conjugate value z*[n] in the time lag domain it is called the signal kernel. Where n is discrete-time index, m is lag index and k is the frequency index. After performing convolution on time lag domain the processed value of k_z[n, m] is transferred from time lag domain to the time-frequency domain by taking Fourier transform to get DWVD in the time-frequency domain.

$$DWVD[n, k] =$$

$$2 \underset{n \rightarrow k}{DFT} k_z[n, m] \quad (7)$$

3.2 Discrete Pseudo-Wigner-Ville distribution

The Wigner-Ville distribution introduced a cross-term because the Fourier transforms on the instantaneous autocorrelation function over the lag, the Wigner-Ville distribution is a non-causal distribution. It is not suitable for real-time signal processing. To minimize the cross term and make suitable for real-time application, applying the Wigner-Ville distribution to

a windowed version of the signal [20]. Setting the time-lag window $G(n, m) = \delta(n)h(m)$. The PWVD of a discrete signal with a finite length lag window is given by

$$DPWVD(n, k) = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} h(m)z(n+m)z^*(n-m)e^{-j\frac{2\pi}{N}km} \quad (8)$$

Where h(m) is a real-valued frequency smoothing window with odd length 2N-1 [21]. Due to the window function, Fourier transforms only consider the signal components in the instantaneous autocorrelation function. Thus the Fourier transform over lag will represent only the frequency components and reduces the cross-term. The effect of the windowing is to smear the signal in a frequency direction without affecting the time resolution. Hanning, Hamming and Gaussian windows are chosen to minimize interference and improve the frequency resolution[22], [23]. The shape and size of these windows are shown in the fig.1. The frequency response of window length 64 is shown in the fig. 2. ENBW value of each window is calculated and given in table-1.

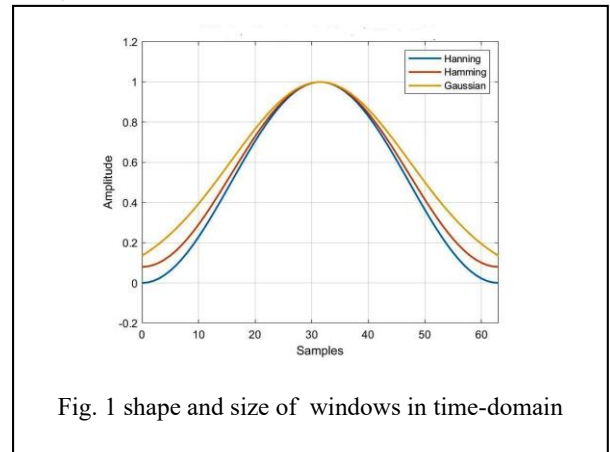


Fig. 1 shape and size of windows in time-domain

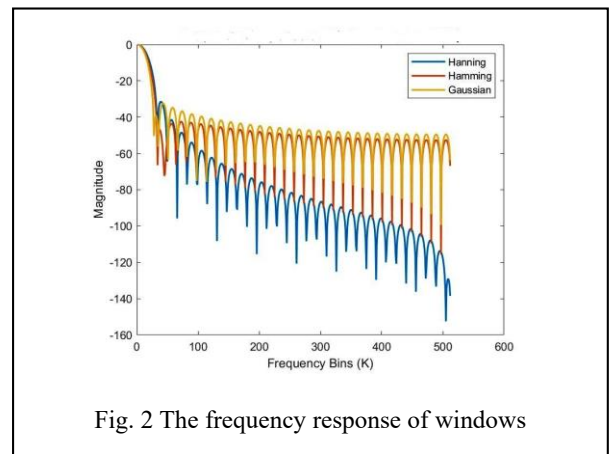


Fig. 2 The frequency response of windows

Window Function	Side lobe level	3db bandwidth	ENBW
Hanning	-32	1.47	1.5
Hamming	-43.5	1.35	1.36
Gaussian	-32.3	1.15	1.23

3.3. Discrete Smoothed Wigner-Ville Distribution

The lag independent smoothed Wigner-Ville distribution is formed by introducing time-dependent window in the Wigner-Ville distribution and is defined as

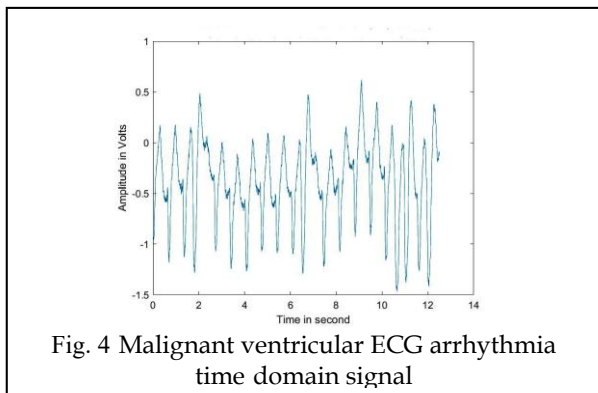
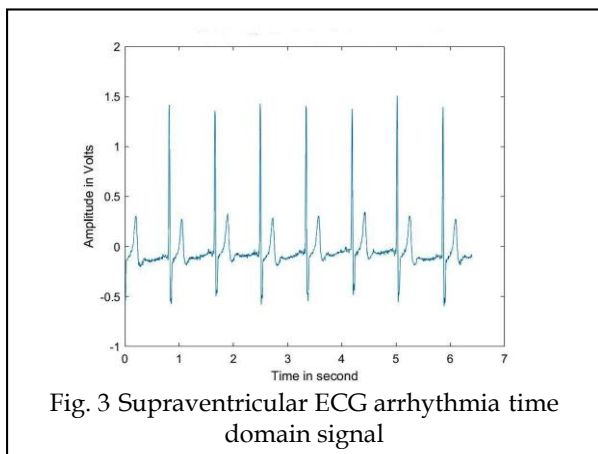
$$DPWVD(n, k) = \sum_{m=-\frac{N}{2}}^{\frac{N}{2}} g[n, m]z(n + m)z^*(n - m)e^{-j\frac{2\pi}{N}km} \quad (9)$$

where $h[n, m]$ is the time-dependent window and is defined over a lag interval compared to the DPWVD in equation(8) the time factor is included in the definition of the time-independent window width $m(n)$ to indicate time-dependent window width for $g[n, m]$ is calculated for all time instants based on lag varying character of the instantaneous autocorrelation function in time lag domain called lag independent smoothed Wigner-Ville distribution. Take DFT back to the time-frequency domain.

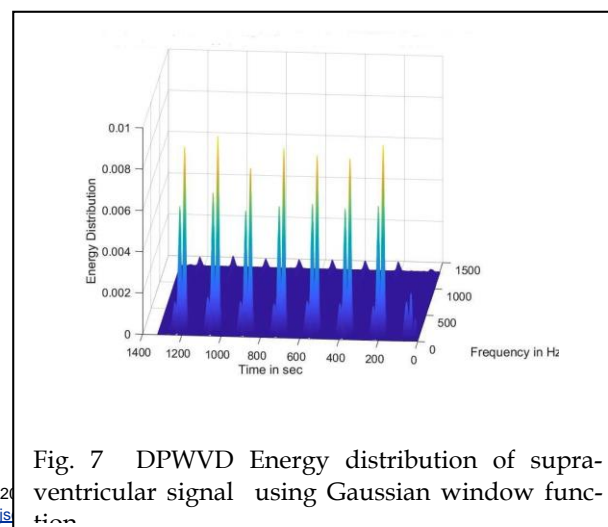
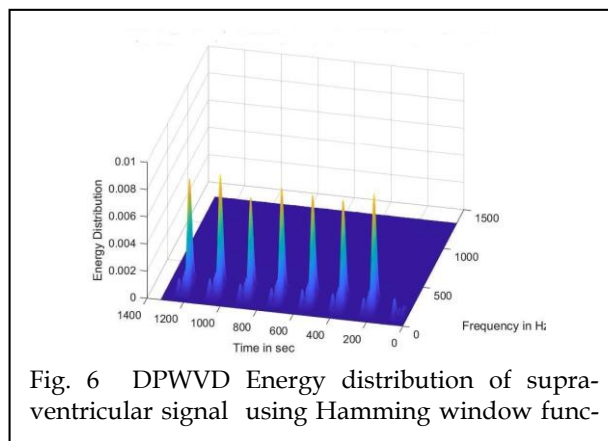
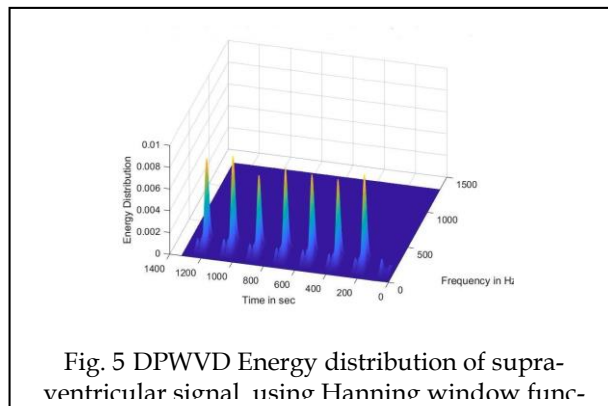
In $g[n, m]$, the time t is transformed into the Doppler domain by taking DFT it is represented as $G[u, m]$. Then perform the instantaneous autocorrelation on the window $G[u]$ in the ambiguity domain called Doppler independent smoothed Wigner-Ville distribution. Take DFT back to the TFD time-frequency domain.

4 Results and discussion

The ECG supraventricular and malignant ventricular signals were pre-processed to remove the various noises and its time-domain signals were shown in the fig. 3 and fig. 4. The pre-processed signals were a real-valued signal converted into an analytical signal. This signal was analyzed using discrete Wigner-Ville distribution. It introduced cross term but it preserves the marginal properties.



The pre-processed supraventricular ECG arrhythmia and malignant ventricular ECG arrhythmia signals were analyzed using discrete pseudo-Wigner-Ville distribution by implementing hanning, hamming and Gauss windows. These windows smeared the discrete pseudo-Wigner-Ville distribution in frequency direction to improve the frequency resolution without affecting the time resolution were shown in the fig.5, fig.6, fig.7, fig.8, fig.9 and fig.10 respectively. The windows were optimized to get frequency resolution and time resolution given in the table -2 and table-3 respectively.



shown in the fig.11, fig.12, fig.13, fig.14, fig.15 and fig.16 respectively. The windows were optimized to get frequency resolution and time resolution given in the table - 2 and 3 respectively.

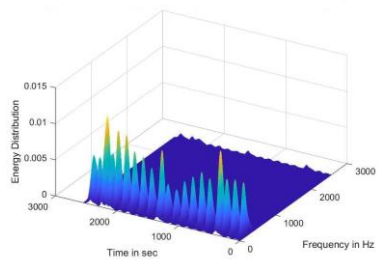


Fig. 8 DPWVD Energy distribution of Malignant ventricular signal using Hanning window function.

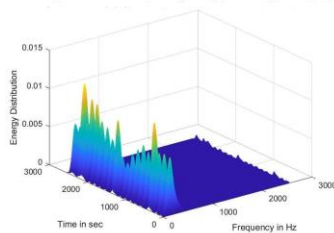


Fig. 9 DPWVD Energy distribution of Malignant ventricular signal using Hamming window function

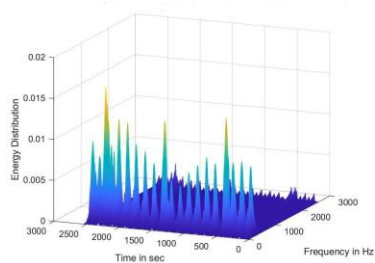


Fig. 10 DPWVD Energy distribution of Malignant ventricular signal using Gaussian window function.

The pre-processed supra-ventricular ECG arrhythmia and malignant ventricular arrhythmia signals were analyzed using smoothed Wigner-Ville distribution by implementing hanning, hamming, Kaiser and Gauss windows. It gives frequency smoothing in time axis and time smoothing in the lag axis as

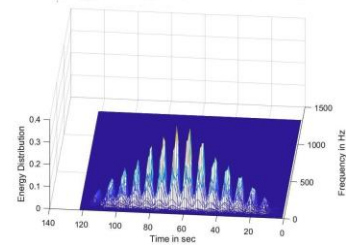


Fig. 11 LI-DSWVD Energy distribution of supra-ventricular signal using Hanning window function

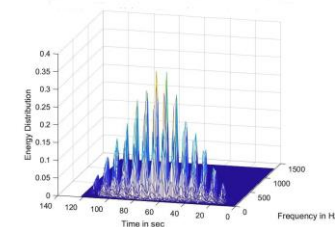


Fig. 12 LI-DSWVD Energy distribution of supra-ventricular signal using Hamming window function

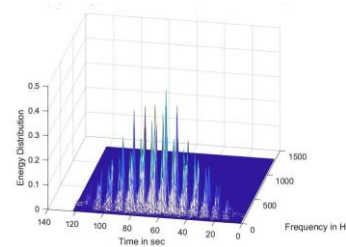


Fig. 13 LI-DSWVD Energy distribution of supra-ventricular signal using Gaussian window function.

The frequency of the supra-ventricular arrhythmia varied for every time instant from 0.059 to 0.11. The three windows gave an energy distribution where the lag independent window decreases the frequency resolution.

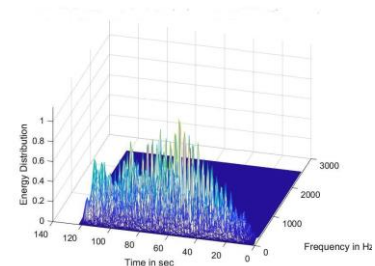


Fig. 14 LI-DSWVD Energy distribution of malignant ventricular signal using Hanning window function.

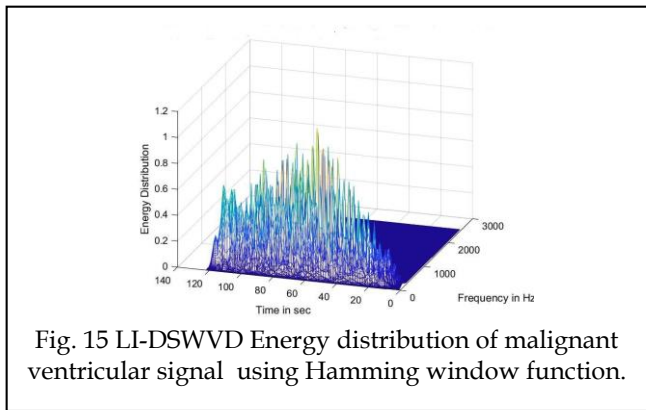


Fig. 15 LI-DSWVD Energy distribution of malignant ventricular signal using Hamming window function.

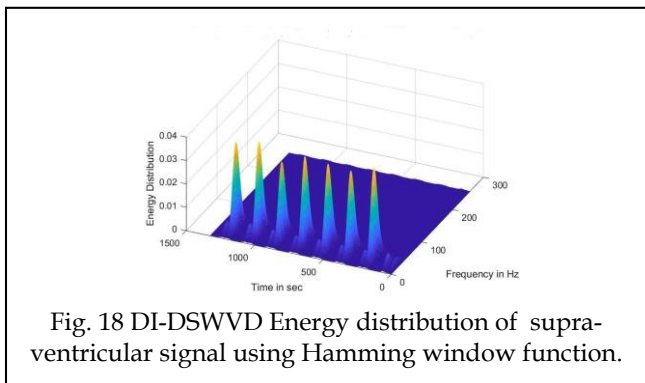


Fig. 18 DI-DSWVD Energy distribution of supra-ventricular signal using Hamming window function.

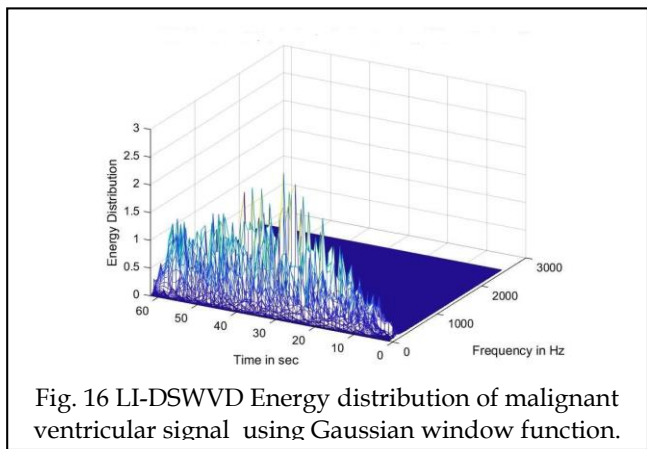


Fig. 16 LI-DSWVD Energy distribution of malignant ventricular signal using Gaussian window function.

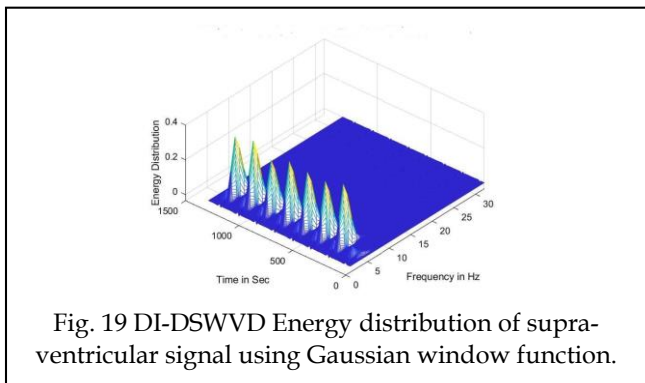


Fig. 19 DI-DSWVD Energy distribution of supra-ventricular signal using Gaussian window function.

In the case of malignant ventricular arrhythmia signal we observed that lag independent discrete smoothed Winger-Ville distribution introduced an un tolerated cross-term in the malignant ventricular arrhythmia energy distribution.

The pre-processed supra-ventricular arrhythmia and malignant ventricular arrhythmia signals were analyzed using Doppler independent smoothed Wigner-Ville distribution by implementing hanning, hamming and Gauss windows. It gives frequency smoothing in lag and time smoothing in the Doppler axis as shown in the fig.17, fig.18, fig.19, fig.20, fig.21 and fig.22 respectively. The windows were optimized to get frequency resolution and time resolution given in the table -2 and 3 respectively

The three windows detected the QRS wave with high energy distribution and time-frequency resolution and the T wave was also detected with low energy distribution.

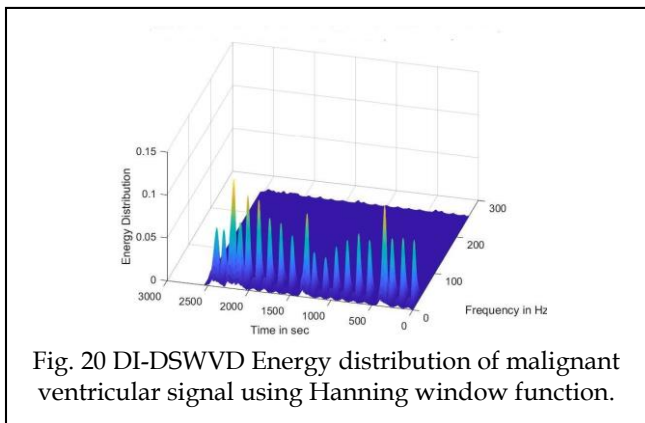
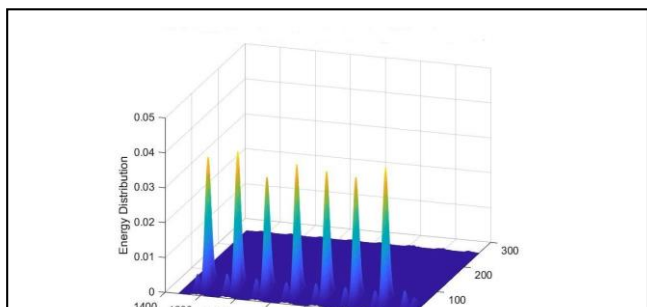
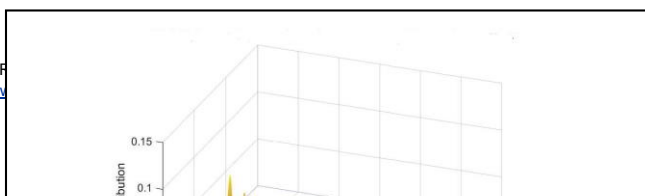


Fig. 20 DI-DSWVD Energy distribution of malignant ventricular signal using Hanning window function.



smoothing, it introduced cross-term in supraventricular and malignant ventricular signals. It preserved all properties. The DPWVD with Gaussian window detected the QRS and T wave multicomponent signal and tracked the energy distribution around the instantaneous frequencies with time frequency resolution and shape changed in QRS complex wave for the supraventricular arrhythmia and malignant ventricular arrhythmia. The Lag independent DSWVD gave an energy distribution where the lag independent window decreased the energy and frequency resolution and introduced cross-terms in malignant ventricular arrhythmia signal. The Doppler independent DSWVD detected QRS and T wave signal and tracked the energy distribution in supraventricular arrhythmia around the instantaneous frequencies with high time-frequency resolution, whereas for malignant ventricular arrhythmia the hanning and Gaussian window detected the QRS and T waves with high time-frequency resolution and clearly shown shape changed QRS wave for supraventricular and malignant ventricular arrhythmia. The result reveals that discrete Wigner-Ville distribution computed in the time lag domain introduced cross-term. The discrete pseudo-Wigner-Ville distribution was computed in the time lag domain and provided low time-frequency resolution. The lag independent discrete smoothed Wigner-Ville distribution window was computed in time lag domain decreased the frequency resolution and the Doppler independent discrete smoothed Wigner-Ville distribution was computed in Doppler lag domain found better resolution performance compared to other proposed distributions.

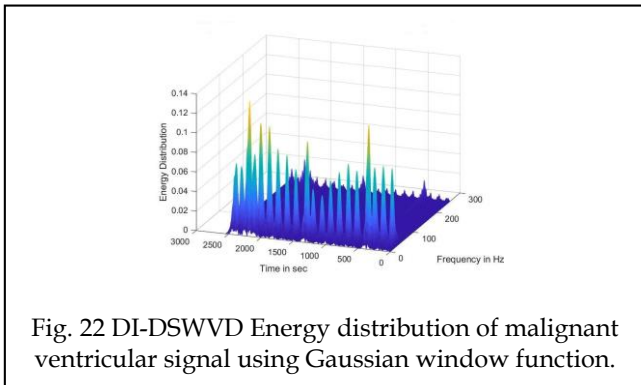


Fig. 22 DI-DSWVD Energy distribution of malignant ventricular signal using Gaussian window function.

Hanning, Hamming and Gauss window resolved P-QRS-T waveform with high energy distribution and time-frequency resolution.

TABLE 2
OPTIMIZED WINDOWWIDTH AND CONTROL PARAMETER FOR SUPRAVENTRICULAR ARRHYTHMIA

Window Function	DPWVD	LI-DSWVD	DI-DSWVD
Hanning	63	121	31
Hamming	63	121	31
Gaussian	63, $\sigma=0.05$	121, $\sigma=0.5$	31, $\sigma=0.05$

TABLE 3
OPTIMIZED WINDOWWIDTH AND CONTROL PARAMETER FOR MALIGNANT VENTRICULAR ARRHYTHMIA

Window Function	DPWVD	LI-DSWVD	DI-DSWVD
Hanning	63	121	31
Hamming	63	121	31
Gaussian	63, $\sigma=0.05$	121, $\sigma=0.5$	31, $\sigma=0.05$

5. Conclusion

The DWVD had no window to control the frequency and time

References

- [1] Braham Barket and Boualem Boashash, " A High Resolution Quadratic Time-Frequency Distribution for Multicomponent Signals Analysis ", IEEE Transactions on Signal Processing ", Vol. 49, No. 10, October 2001.
- [2] Boualem Boashash and Saman S. Abeysekera, " Two Dimensional Processing of Speech and ECG signals Using The Wigner-Ville Distribution", Application of Digital Image Processing IX SPIE, 1986.
- [3] M. Wacker and H. Witte, " Time-Frequency Techniques in Biomedical Signal Analysis, A Tutorial Review of Similarities and Differences", Methods Inf Med 2013; 52: 279-296, doi: 10.3414/ ME 12-01-0083.
- [4] S.Sivakumar and D. Nedumaran, "Discrete Time-Frequency Signal Analysis and Processing Techniques for Non-Stationary Signals", Journal of Applied Mathematics and Physics, 2018, Vol. 6, pp. 1916-1927. ISSN online:2327-4379, doi: 10.4236/jamp 2018.69163.
- [5] S.Gokhun Tanyer, Gorkem Cipli and Peter Driessen, "High Resolution Time-frequency Analysis of Non-Stationary Signals", Proceedings of the 4th International Conference of Control, Dynamic Systems, and Robotics (CDSR'17) Toronto, Canada - August 21 - 23, 2017, Paper No.126. doi:10.11159/cdsr17.126.
- [6] Boualem Boashash and Peter J. Black, "An Efficient Real-Time Implementation of the Wigner-Ville Distribution", IEEE Transactions on Acoustics, Speech and Signal Processing, vol. ASSP-35, No 11 November 1987.
- [7] Zhongguo Liu1, Changchun Liu1, Boqiang Liu1, Yangsheng Lv2, Yinsheng Lei2 and Mengsun Yu1, " Self Spectrum Window Method in Wigner-Ville Distribution", Proceedings of the 2005 IEEE ,Engineering in Medicine and Biology 27th Annual Conference ,Shanghai, China, September 1-4, 2005
- [8] T.A.C.M. Claasen and W.F.G. Mecklenbrauker, "The Wigner Distribution - A Tool For Time-Frequency Signal Analysis", Philips Journal of Research. Vol.35, No. 6, pp. 372-389, 1980.
- [9] S. E. Qian, D. P. Chen, " Joint time-frequency analysis: methods and applications", Upper Saddle River, NJ: Prentice Hall PTR, 1996. pp.

203-235.

- [10] F. S. Yang, Y. S. Lu, Biomedical signal processing and recognition, Tianjin: Tianjin Science & Technology Translation & Publishing Co., 1997, pp. 224-284.
- [11] Z. Lin, and J. D. Chen, "Advances in time-frequency analysis of biomedical signals," *Crit Rev Biomed Eng*, vol.24, no. 1, pp. 1-72 Jan 1996.
- [12] Y. S. Yan, C. C. Poon, and Y. T. Zhang, "Reduction of motion artifact in pulse oximetry by smoothed pseudo Wigner-Ville distribution," *J Neuroengineering Rehabil*, vol. 2, no. 1, pp. 3, Mar 2005
- [13] S. Ghofrani, A. Ayatollahi, and M. B. Shamsollahi, "Wigner-Ville distribution and Gabor transform in Doppler ultrasound signal processing," *Biomed Sci Instrum*, vol. 39, pp. 142-147, Jan. 2003.
- [14] E. Pereira, M. A. Custaud, J. Frutoso, L. Somody, C. Gharib, and J. O. Fortrat, "Smoothed pseudo Wigner-Ville distribution as an alternative to Fourier transform in rats," *Auton Neurosci*, vol. 87, no. 2-3, pp. 258-67, Mar 2001
- [15] Andria G.M. Savino, " Interpolated Smoothed Pseudo Wigner-Ville Distribution for Accurate Spectrum Analysis", *IEEE Trans. on Instrumentation and Measurement*. 45(4): 818-823
- [16] Roshan Joy Marits U. Rajendra Acharya and Hojjat Adeli, " Current Methods in Electrocardiogram Characterization", *Computers in Biology and Medicine*, doi:10.1016/j.compbimed.2014.02.012, 2014 Elsevier
- [17] A. Dliou, R.Latif and M.Laaboubi, " Arrhythmia ECG Signal Analysis using Non Parametric Time-Frequency Techniques", *International Journal of Computer Applications* (0975 - 8887) Vol. 41 - N0.4. March 2012.
- [18] Allam Mousa and Rashid Saleem, " The Shortage of WVD in Analyzing Abnormal ECG Signal", *IEEE Symposium on Industrial Electronics and Applications (ISIEA 2010)*, October 3-5, 2010, Penang, Malaysia.
- [19] Physiobank, Physionet, Physiologic Signal Archives for Biomedical Research http://www.physionet.org/physio_bank.
- [20] Sanjit K. Dash and G. Sasibhushana Rao, " A Comparative Study of Pseudo-Wigner-Ville Distribution (PWVD), WVD and STFT in ECG Signal Analysis", *International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering*", vol. 6, Issue 9, pp. 6899-6910, September 2017, doi:10.15662/IJAREEIE.2017.0609038.
- [21] Ahmad ZuriSha' Ameri and Boualem Boashash, "The Lag Windowed Wigner-Ville Distribution: An Analysis Method for HF Data Communication Signals", *FurnalTeknologi* 30 ID: 33-54 (C) Universiti Teknologi Malaysia
- [22] K.M.M. Prabhu and R. Shanmugasundaram, "Fast Algorithm for Pseudo-discrete Wigner-Ville Distribution using Moving Discrete Hartley Transform", *IEEE Proc.-Vis. Image Signal Processing*, Vol. 143, No. 6, December 1996.
- [23] Pierre Wickranarachi, "Effects of Windowing on the Spectral Content of a Signal", *Sound and Vibration*, January 2003.